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Performance analysis of two-stage TECs (thermoelectric coolers) using a three-dimensional heat-electricity coupled model

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A B S T R A C T

This work for the first time uses a three-dimensional multi-physics model to optimize the performance of three kinds of two-stage TECs, connected electrically in series, in parallel, and separated, respectively. The optimizations are performed for the two-stage TEC with 30 thermoelectric elements. The number ratio and current ratio are searched to reach the optimal cooling capacity, COP, and maximum temperature difference, respectively. A marked three-dimensional temperature distribution is observed for the two-stage TEC with number ratio larger or smaller 1.00. In addition, temperature-dependent material properties are proven to be extremely important for predicting the two-stage TEC performance. Therefore, thermal resistance models extensively adopted in the previous two-stage TEC studies can not predict the two-stage TEC performance accurately because they assume the one-dimensional temperature distribution and constant material properties. The results also show that the thermoelectric element number on the hot stage should be larger than that on the cold stage for improving the cooling capacity and COP, and the optimal number ratio is found to be about 1.73–2.33 for the series configuration. The performance can be further improved by supplying a higher current to the hot stage, and the optimal current ratio ranges from 1.50 to 2.00.

1. Introduction

Recently, along with the rapid progress of thermoelectric materials, the research and development of thermoelectric devices have attracted more and more attention because the devices do not use any moving parts and environmentally harmful fluids [1]. The thermoelectric devices can be divided into two different groups: TEGs (thermoelectric generators) [2–4] and TECs (thermoelectric coolers) [5,6]. TECs pump heat from a low temperature heat source to a high temperature one using Peltier effect when the current is supplied to TECs, and they have been widely used for the cooling purpose of electronic devices such as CPU, infrared detector and sensor, charge-coupled device, ice-point reference in thermocouple, and refrigerator [7–10].

There are two parameters for evaluating the TEC performance, one is the cooling capacity defined as the heat adsorbed by the cold end of the TEC, the other one is the COP (coefficient of performance) defined as the ratio of cooling capacity to electrical power consumed by the TEC [10]. The TEC performance is determined by thermoelectric material properties (figure-of-merit, *ZT*), TEC geometric structure, and TEC operating condition (applied current and temperature difference between the hot and cold ends) [1,7,10]. The convectional TEC has one-stage configuration. Due to the performance limits of thermoelectric materials, a commercially available one-stage TEC can provide at most a 70 K maximum temperature difference, when its hot end remains at room temperature [11]. Therefore, when a large temperature difference is required for some special applications, the one-stage TEC will not be qualified. Furthermore, a larger temperature difference also reduces the COP of the one-stage TEC significantly, which hence causes a poor operational thermal economics.

In order to obtain larger temperature difference and to improve the COP, a two- or multi-stage TEC design was proposed [12], where two or more thermoelectric modules are stacked one on top of the other. Followed then, a number of studies [13–34] has proven that the two- or multi-stage TEC can provide a temperature difference larger than 100 K due to additivity of temperature difference produced by each stage. For two-stage TECs, there are three typical approaches to supply the electric current to each stage: in series, in parallel, and separated. Thus, except material properties, the

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cooling capacity, COP and maximum temperature difference of two-stage TECs is expected to be related to the number ratio of thermoelectric elements between the two stages, and the applied current of each stage.

Table 1 summarizes the recent numerical investigations on two-stage TEC [11,13–34]. Most of the models adopted so-called thermal resistance model or zero-dimensional model [11,13–33], in which two algebraic energy equations are constructed only on the hot and cold ends of each stage based on the energy conservation law. This kind of models can directly derive analytical expressions of the cooling capacity and COP. However, the disadvantages are also obvious. First, constant or temperature-averaged material properties were used because internal heat transfer and electricity transport were not modeled. Constant Seebeck coefficient means ignored Thomson effect. Huang et al. [35], Chen et al. [36] and Wang et al. [10] have confirmed that TEC performance could be improved not only by increasing the figure-of-merit of thermoelectric materials but also by taking the Thomson effect into account. Second, Joule heat (internal heat source) is split into two equal parts to be distributed to hot and cold ends of each stage. Third, the p- and n-type semiconductors are usually simplified as an entire individual bulk phase so that the differences in the thermal and electric behaviors between them are not possible to be distinguished and evaluated. Recently, Liu and Wen [34] used a one-dimensional heat conduction model to investigate the two-stage TEC performance. They adopted constant and temperature-dependent Seebeck coefficient (quadratic polynomial and log-linear) and found that the model with the quadratic polynomial Seebeck coefficient can predict the performance more accurately. Their results also confirmed that Thomson heat can enhance the cooling performance of two-stage TEC under specific conditions.

To success a thermal management of any electronic component, an accurate determination of TEC performance is necessary. Minnich et al. [1] proposed that Seebeck coefficient, thermal conductivity and electric conductivity of most thermoelectric materials are all strongly temperature-dependent. The thermal conductivity affects the amount of heat transferred from the hot end to cold end by Fourier heat conduction remarkably, and the electric conductivity determines the total electric resistance of p–n junction. Thus, the heat amount and the total electric resistance, which are predicted by constant and temperature-dependent material properties, differ from each other significantly when the TEC operates at large temperature differences and/or at high applied currents [10]. Furthermore, the thermal resistance model and the one-dimensional heat conduction model assume that the current densities uniformly distribute within the p- and n-type semiconductors, so that electric potential equation is ignored. Our previous [10,37–39] and Cheng and Huang's [40] studies have shown that three-dimensional distributions of the temperature and current density are observed for high temperature differences and large currents, especially when the heat loss of the TEC to the ambient can not be ignored.

This paper uses a general, three-dimensional, and heat-electricity coupled model developed in our previous study [10] to investigate the two-stage TEC performance. The model accounts for

Table 1
A review of previous numerical models of two-stage TEC.

Authors	Model	Thomson effect	Current density	Material properties	Design configuration	Separate p and n semiconductors	Temperature distributions in p and n semiconductors
Xuan, 2002 [11]	Energy equilibrium equations at hot and cold ends	No	Uniform current density, $J = I/A$	Constant	Series, parallel, separated	No	No
Xuan et al., 2002 [13]				Constant[b]	Series, parallel, separated		
Xuan et al., 2002 [14]				Constant[a]	Series, separated		
Chen et al., 2002 [15]				Constant	Series		
Xuan, 2003 [16]				Constant[b]	Series, separated		
Lai et al., 2004 [17]				Constant	Series		
Yang et al., 2004 [18]				Constant	Separated		
Chen et al., 2005 [19]				Constant	Series		
Cheng and Shih, 2006 [20]				Constant[a]	Series, separated, parallel		
Cheng and Shih, 2006 [21]				Constant[b]	Series, separated		
Cheng and Shih, 2006 [22]				Constant[a]	Series, separated, parallel		
Yu et al., 2007 [23]				Constant	Separated		
Chen et al., 2007 [24]				Constant	Series		
Li et al., 2008 [25]				Constant	Separate		
Chen et al., 2008 [26]				Constant	Series		
Chen et al., 2009 [27]				Constant	Series		
Meng et al., 2009 [28]				Constant	Series		
Meng et al., 2010 [29]				Constant	Series		
Meng et al., 2010 [30]				Constant	Series		
Razani et al., 2012 [31]				Constant	Separated		
Olivares-Robles et al., 2012 [32]				Constant	Separated		
Rao and Patel, 2013 [33]				Constant[b]	Separated, series		
Liu and Wen, 2011 [34]	One-dimensional heat conduction model with Joule heat and Thomson heat as internal heat sources	Yes		Temperature-dependent (Seebeck coefficient)	Series, parallel, separated		Yes

[a] Material properties are calculated by temperature-dependent property formulations $\alpha = f_1(T)$, $\rho = f_2(T)$, $\lambda = f_3(T)$, with a characteristic temperature $T_m = (T_H + T_L)/2$, so properties are still treated to be the constants in essential.

[b] Material properties are calculated by temperature-dependent property formulations $\alpha = f_1(T)$, $\rho = f_2(T)$, $\lambda = f_3(T)$, with a characteristic temperature $T_{m1} = (T_H + T_m)/2$, $T_{m2} = (T_m + T_L)/2$, so properties are still treated to be the constants in essential.

all thermal and electric phenomena occurred in the TEC, including Seebeck effect, Peltier effect, Thomson effect, Joule heating, Fourier heat conduction. The performances of the two-stage TEC with the constant and temperature-dependent material properties are first compared. Then the model with temperature-dependent material properties is used to optimize the cooling capacity and COP of the three kinds of two-stage TECs, which are connected electrically in series, in parallel, and separated, respectively. The number ratio of thermoelectric elements between hot and cold stages for the series configuration, as well as the current ratio of hot stage to cold stage for separated configuration, are optimized. The maximum temperature difference for the three configurations is analyzed.

2. Structure of two-stage TEC

The schematics of two-stage TECs with electrically connected in series, in parallel, and separated, respectively, are shown in Fig. 1. The TEC is composed of a top stage (cold stage) with m thermoelectric elements and a bottom stage (hot stage) with n thermoelectric elements (Fig. 2). Total number of thermoelectric elements is $N = m + n$, thus, a number ratio of thermoelectric elements between the two stages can be defined as $r = n/m = n/(N - n)$. Each thermoelectric element consists of a p-type semiconductor column, a n-type semiconductor column, three metal connectors, and two ceramic plates. The p- and n-type semiconductor columns have the same rectangular cross-sectional area of $L_1 \times L_2$ and their separate distance is L_3. The thicknesses of p- and n-type semiconductor columns, metal connectors, and ceramic plates are H_1, H_2, and H_3.

The thermoelectric element converts electricity to heat by Peltier effect, however, its cooling capacity is not as strong as the Peltier heat, it should be determined by the combination of Peltier heat, Fourier's heat conduction, Joule heating, and Thomson heat. Therefore, the temperature difference between the hot and cold ends and the applied current affect the cooling capacity of the thermoelectric element. For the two-stage TEC, the heat is transferred in parallel in each stage, while in series between two stages. Consequently, with a fixed temperature difference, the total cooling capacity of the two-stage TEC is strongly dependent on the number of the thermoelectric elements of each stage and the applied current flowing through each stage.

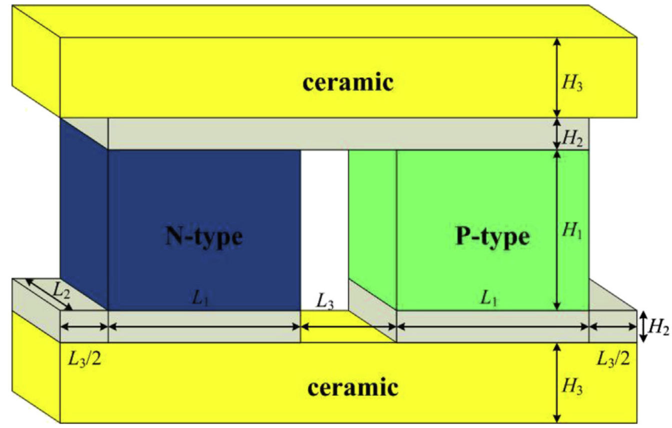

Fig. 2. Schematic of thermoelectric element.

This work firstly searches for the optimal number ratio at different temperature differences for the series configuration to

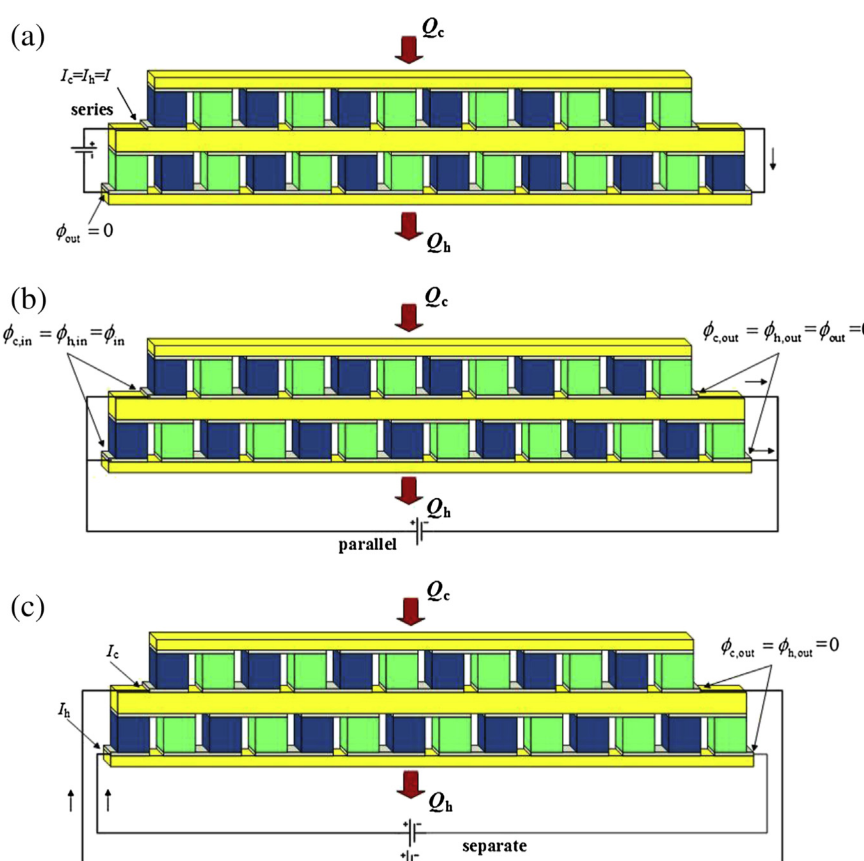

Fig. 1. Schematics of three kinds of two-stage TECs: (a) electrically connected in series; (b) electrically connected in parallel; (c) electrically connected separated.

obtain the maximum cooling capacity and COP of two-stage TEC. Then, the cooling capacities and COPs are compared for the series and parallel configurations at the same optimal number ratio. Finally, the optimal current ratio is determined for the separated configuration. Here, the current ratio is defined as $t = I_h/I_c$, where I_h is the current flowing through the hot stage and I_c is the current flowing through the cold stage.

3. Model

The present heat-electricity coupled model includes the energy equations and the electric potential equations. The model adopts the following assumptions: the two-stage TEC operates at steady state; the n- and p-type semiconductor columns are modeled as two different parts; semiconductor properties are temperature-dependent; heat loss to the ambient is ignored; the contact thermal and electrical resistances are ignored. The energy equations are as follows:

$$\nabla(\lambda_{\rm p}\nabla T) + \frac{J^2}{\sigma_{\rm p}} - \beta_{\rm p}\vec{J}\cdot\nabla T = 0 \tag{1a}$$

$$\nabla(\lambda_{\rm n}\nabla T) + \frac{J^2}{\sigma_{\rm n}} - \beta_{\rm n}\vec{J}\cdot\nabla T = 0 \tag{1b}$$

$$\nabla(\lambda_{\rm conn}\nabla T) + \frac{J^2}{\sigma_{\rm conn}} - \beta_{\rm conn}\vec{J}\cdot\nabla T = 0 \tag{1c}$$

$$\nabla(\lambda_{\rm cer}\nabla T) = 0 \tag{1d}$$

The first, second, and third terms of the left side of Eq. (1) describe the Fourier heat conduction, Joule heating, and Thomson effect, respectively. The subscripts p, n, conn, and cer denote the p- and n-type semiconductor columns, the metal connector, and the ceramic plate, respectively. T is the temperature, λ is the thermal conductivity, σ is the electric conductivity, β is the Thomson coefficient expressed by $\beta = T({\rm d}\alpha/{\rm d}T)$, α is the Seebeck coefficient. The ceramic plate is electrically insulated, Joule heat and Thomson heat can be safely ignored. J is the local current density and may be distributed non-uniformly within the TEC, hence, the electric potential equations are needed for the p- and n-type semiconductors and the metal connectors, or

$$\nabla\cdot\left(\sigma_{\rm p}\left(\nabla\phi - \alpha_{\rm p}\nabla T\right)\right) = 0 \tag{2a}$$

$$\nabla\cdot\left(\sigma_{\rm n}\left(\nabla\phi - \alpha_{\rm n}\nabla T\right)\right) = 0 \tag{2b}$$

$$\nabla\cdot\left(\sigma_{\rm conn}\left(\nabla\phi - \alpha_{\rm conn}\nabla T\right)\right) = 0 \tag{2c}$$

where ϕ is the electric potential. The current density is related to the electric potential by the following equation:

$$\vec{J} = \sigma(-\nabla\phi + \alpha\nabla T) \tag{3}$$

The boundary conditions are described as follows. The isothermal conditions are applied to the cold end of the cold stage and to the hot end of the hot stage, and the adiabatic conditions are applied to the side surfaces of the two-stage TEC. The temperature, heat flux, electric potential, and current density, are continuous on the interface between any two bulk materials. It is noted that the Peltier heat will be generated on the interface between semiconductor column and metal connector, thus, the heat flux continuity is modified as

$$\begin{aligned}&\left(-\lambda_{\rm conn}\frac{\partial T_{\rm conn}}{\partial n} + \alpha_{\rm conn}T_{\rm conn}J_{\rm n}\right)_{\rm interface}\\ &= \left(-\lambda_{\rm semi}\frac{\partial T_{\rm semi}}{\partial z} + \alpha_{\rm semi}T_{\rm semi}J_{\rm n}\right)_{\rm interface}\end{aligned} \tag{4}$$

where the subscript semi denotes n- or p-type semiconductor column.

For the series configuration, the currents are the same for the cold and hot stages, thus, we have: $I_c = I_h = I$, the electric potential at the outlet of the two-stage TEC are assumed to be $\phi_{\rm out} = 0$, the electric potential at the inlet of the two-stage TEC, $\phi_{\rm in}$, is solved by the model, and the electric potential difference through the two-stage TEC is calculated by $V = \phi_{\rm in} - \phi_{\rm out}$. For the parallel configuration, the electric potential differences are the same for the cold and hot stages, therefore, we have: $\phi_{c,{\rm in}} = \phi_{h,{\rm in}} = \phi_{\rm in}$ at the inlet of each stage, $\phi_{c,{\rm out}} = \phi_{h,{\rm out}} = \phi_{\rm out} = 0$ at the outlet of each stage, and $V = \phi_{\rm in} - \phi_{\rm out}$, the current of each stage can be solved by the model and the total current is calculated by $I = I_c + I_h$. For the separated configuration, the currents of the cold and hot stages, (I_c and I_h) and $\phi_{c,{\rm out}} = \phi_{h,{\rm out}} = 0$ are given, $\phi_{c,{\rm in}}$ and $\phi_{h,{\rm in}}$ are solved by the model, thus, the electric potential differences of the cold and hot stages are calculated by $V_c = \phi_{c,{\rm in}} - \phi_{c,{\rm out}}$ and $V_h = \phi_{h,{\rm in}} - \phi_{h,{\rm out}}$, respectively.

Except the inlet and outlet, the current can not flow out of the two-stage TEC, thus, we have

$$\vec{J}\cdot\vec{n} = 0 \tag{5}$$

Cooling capacity of the two-stage TEC, $Q_{c,c}$, is defined as the heat absorbed by the cold end of the cold stage. The COP is defined as the ratio of cooling capacity to electric power expressed as

$${\rm COP} = \frac{Q_{c,c}}{P} = \frac{Q_{c,c}}{IV} \tag{6}$$

where P is the electric power, I is the current, V is the electric potential difference. For separated configuration, $P = P_c + P_h = I_cV_c + I_hV_h$.

The geometry of the two-stage TEC adopted for simulations has $H_1 = 1.0$ mm, $H_2 = 0.1$ mm, $H_3 = 0.5$ mm, $L_1 = 1.5$ mm, $L_2 = 1.2$ mm, and $L_3 = 0.2$ mm. $Bi_2(Te_{0.94}Se_{0.06})_3$ and $(Bi_{0.25}Sb_{0.75})Te_3$ with temperature-dependent properties are chosen for the n- and p-type semiconductors due to their excellent thermoelectric properties [41]. The copper is selected as the connector with constant properties due to its weak thermoelectric effect. The material properties of the semiconductors and copper can be found in our previous work [10]. The thermal conductivity of the ceramic plate is 35.3 W m^{-1} K^{-1} [42]. The TEC model has been validated in our previous works for the steady-state operation [10] and the transient operation [37], more details can be found in Refs. [10,37].

4. Results and discussion

4.1. Arrangement of thermoelectric elements on cold and hot stages

When the cold and hot stages have different numbers of thermoelectric elements, the arrangement of thermoelectric elements may affect the performance of the two-stage TEC. The tests are performed for a two-stage TEC with series configuration. The cold stage has two thermoelectric elements and the hot stage has four elements. Five different arrangements (Fig. 3) are compared at the same operating conditions of $T_{h,h} = T_{c,c} = 300$ K and $I = 8$ A. At this applied current the five arrangements have the best cooling capacity. Table 2 shows the cooling capacity and COP for the five arrangements. The arrangement (d) has the highest cooling capacity of 1.475 W and COP of 0.374, and then followed by arrangements (a)–(c), and (e). The arrangement (e) has the lowest cooling

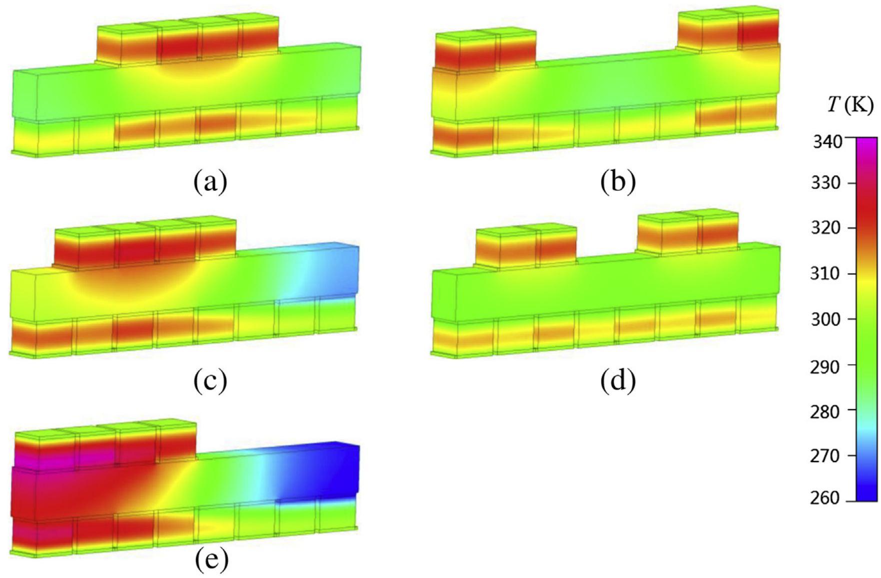

Fig. 3. Five arrangements of thermoelectric elements of two-stage TEC.

capacity of 1.251 W and COP of 0.300, which are 15.2% for the cooling capacity and 19.8% for the COP lower than those of the arrangement (d). The result confirms the effect of the thermoelectric element arrangement on the performance of the two-stage TEC.

Fig. 4 shows the temperature distributions on the bottom of the ceramic plate sandwiched between the hot and cold stages for the five arrangements. The number of the thermoelectric elements of the cold stage is less than that of the hot stage, the cold stage only partly covers the ceramic plate. Thus, when the hot end of the cold stage liberates the heat to the ceramic plate, the temperature of the ceramic plate is higher under the cold stage than the other place, which inevitably causes a non-uniform temperature distribution on the cold end of the hot stage. This result indicates that each thermoelectric element of the hot stage operates at different temperature difference, and hence has different cooling capacity. The arrangement (d) has more uniform temperature distribution on the cold end of the hot stage and the maximum temperature difference is only 3.8 K, because its two thermoelectric elements of the cold stage are arranged more uniformly compared with the other four arrangements. However, the maximum temperature difference is increased to 60.5 K for the arrangement (e). Thus, the performance difference for the five arrangements can be attributed to the temperature uniformity of the cold end of the hot stage.

It is worth noting that previous thermal resistance models [11,13–33] and one-dimensional heat conduction model [34] ignored the three-dimensional temperature distribution within the two-stage TEC and only accounted for the temperature variation along the semiconductor height direction. The assumption of one-dimensional temperature distribution is basically reasonable for the one-stage TEC, when constant material properties are adopted and the heat loss of the TEC to the ambient can be ignored. However, the present result shows that for the two-stage TEC, even though the heat loss is ignored, the three-dimensional temperature distribution is still very remarkable due to the arrangement of the thermoelectric elements. Thus, a three-dimensional model is indeed necessary to accurately predict the performance of the two-stage TEC.

Table 2
Cooling capacity and COP of two-stage TECs with five different arrangements.

Arrangement	I (A)	V (V)	Q_c (W)	COP
a	8.00	0.501	1.413	0.353
b	8.00	0.501	1.413	0.353
c	8.00	0.505	1.375	0.340
d	8.00	0.493	1.475	0.374
e	8.00	0.520	1.251	0.300

4.2. Comparison between constant and variable material properties

The performances of the two-stage TEC, predicted by the models with constant and temperature-dependent material properties, are compared to reveal the effect of material properties. The simulations are carried out for a two-stage TEC with $N = 30$ and $r = 2$. The cold end temperature of the cold stage is assumed to be $T_{c,c} = 240$ K, and the hot end temperature of the hot stage is $T_{h,h} = 300$ K, thus, the TEC operates at a temperature difference of 60 K. For the constant material properties, the reference temperature is assumed to be 300 K.

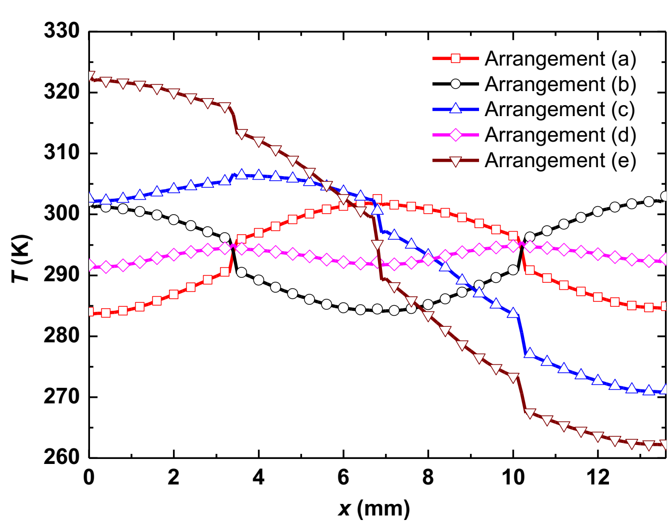

Fig. 4. Temperature distributions along x-direction on the bottom of the ceramic plate between the hot and cold stages for the five arrangements.

The internal temperature distributions along the height direction of the two-stage TEC predicted by the both models are shown in Fig. 5a. At a low current of $I = 3$ A, only a small amount of Joule heat (J^2/σ) and Thomson heat ($-\beta J(dT/dz)$) is produced, the material properties have a small effect on the temperature distribution. As the current increases, the Joule heat and Thomson heat are both increased and the Joule heat is increased faster because it is proportional to I^2. As a result, the difference of temperature distributions between the two models occurs. At $I = 8$ A, the temperature within the TEC seems to be lower than 300 K everywhere. Since the electric conductivities of $Bi_2(Te_{0.94}Se_{0.06})_3$ and $(Bi_{0.25}Sb_{0.75})Te_3$ have a negative temperature effect, the constant-properties model underestimates the electric conductivity of $Bi_2(Te_{0.94}Se_{0.06})_3$ and $(Bi_{0.25}Sb_{0.75})Te_3$ at $I = 8$ A, and hence overestimate the Joule heat and resulting temperature distribution. However, at $I = 12$ A, the temperature is larger than 300 K in most regions of the TEC, so the constant-properties model underestimate the Joule heat and the temperature distribution.

The difference of temperature distributions predicted by the two models leads directly to the difference of cooling capacities, as shown in Fig. 5b. At small currents of $I < 5$ A, the two models predict almost the same cooling capacity. At 5 A $< I <$ 12 A, the constant-properties model underestimates the cooling capacity, however, at $I > 12$ A, it overestimates the cooling capacity. This result can be explained by the fact that the overestimated temperature distribution enhances the Fourier heat conduction, which transfers more heat to the cold end of the cold stage, and hence reduces the cooling capacity, and vice versa.

4.3. Optimal number ratio of thermoelectric elements for series configuration

The performance of the two-stage TEC with series configuration is compared for various r. The total number of the elements is $N = 30$, the hot end temperature of the hot stage is $T_{h,h} = 300$ K, the temperature difference is assumed to be $\Delta T = T_{h,h} - T_{c,c} = 0$, 60, and 80 K.

Fig. 6 shows the cooling capacity and COP of the two-stage TEC with various r at $\Delta T = 0$ K. For each r, there exists an optimal applied current, I_{opt}, at this current the TEC has the maximum cooling capacity, $Q_{c,c,max}$. As r increases from 0.25 to 29, $Q_{c,c,max}$ first increases and then decreases. Optimal r = 1.73 and 2.00 is observed, the corresponding $Q_{c,c,max}$ and I_{opt} are 7.14 W and 7.86 A for r = 1.73, 7.19 W and 8.00 A for r = 2.00, respectively. Increase in r also extends the effective working range of the applied current, for example, the maximum current is 2.9 A for $r = 0.25$, it increased to 5.7 A for $r = 0.5$, and to 16.3 A for $r = 29.00$. Different from $Q_{c,c}$, the COP monotonously decreases as the applied current increases at

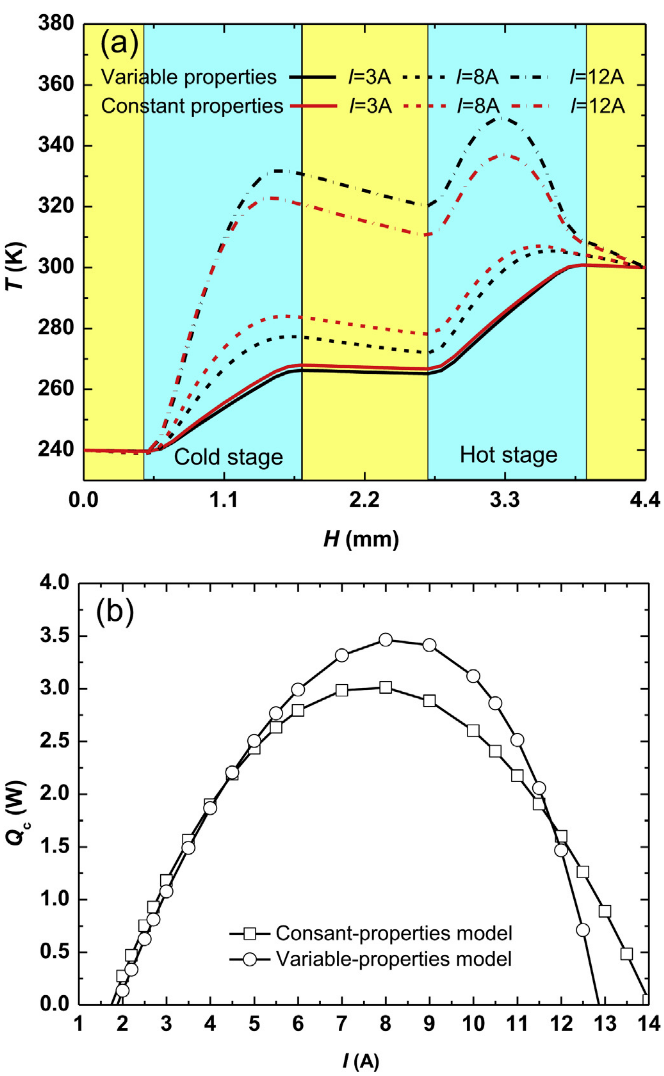

Fig. 5. Performance comparison of two-stage TEC with constant and variable material properties.

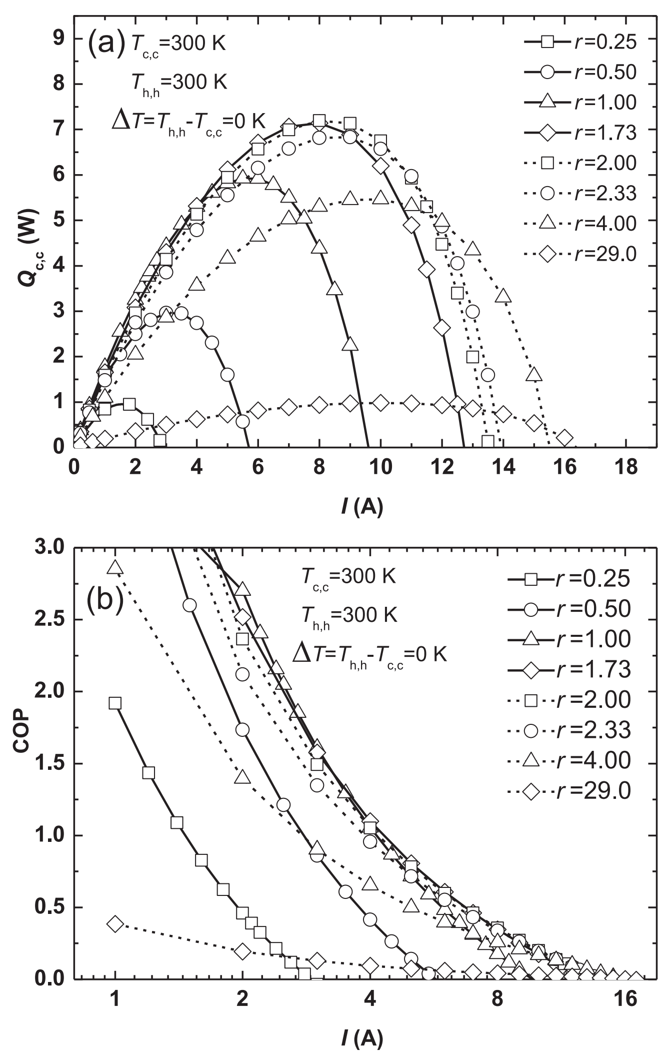

Fig. 6. Cooling capacities and COPs of two-stage TECs with various r.

$\Delta T = 0$ K, and $r = 1.00$ has the optimal COP at $I < 3.6$ A, $r = 1.73$ has the optimal COP at 3.6 A $< I <$ 11 A, and $r = 2.00$ has the optimal COP at $I >$ 11 A (Fig. 6b). As a result, with consideration of both cooling capacity and COP, the optimal r is 1.73 when the two-stage TEC operates at $\Delta T = 0$ K.

The temperature distribution along the height direction of the two-stage TEC for various r at $\Delta T = 0$ K is shown in Fig. 7. For the two-stage TEC, the heat adsorbed by the cold end of the hot stage, $Q_{c,h}$, is the sum of the heat adsorbed by the cold end of the cold stage, $Q_{c,c}$, and the electric power P_c consumed by the cold stage, or $Q_{c,h} = Q_{c,c} + P_c$. For $r < 1$, the thermoelectric element number of the cold stage is larger than that of the hot stage. Thus, each thermoelectric element of the hot stage must adsorb more heat than that of the cold stage. Fig. 7a shows that for $r < 1$, $T_{c,c}$ is smaller than $T_{h,c}$ (the hot end temperature of the cold stage), however, $T_{c,h}$ (the cold end temperature of the hot stage) is larger than $T_{h,h}$. For a thermoelectric element, when the hot end temperature is larger than the cold end temperature, Fourier heat conduction transfers back a part of heat from the hot end to the cold end, and hence reduces its cooling capacity. However, when the hot end temperature is lower than the cold end temperature, Fourier heat conduction has a positive effect on the cooling capacity because it transfers a part of heat from the cold end to the hot end, enhancing the cooling capacity. Thus, for $r < 1$, $T_{c,c} < T_{h,c}$ reduces the cooling capacity of each thermoelectric element of the cold stage, while $T_{c,h} > T_{h,h}$ enhances the cooling capacity of each thermoelectric element of the hot stage. As r increases, the thermoelectric element number of the cold stage decreases, the cooling capacity of each thermoelectric element of the cold stage should be enhanced, while the cooling capacity of each thermoelectric element of the hot stage should be reduced. Thus, the temperature difference of the cold stage

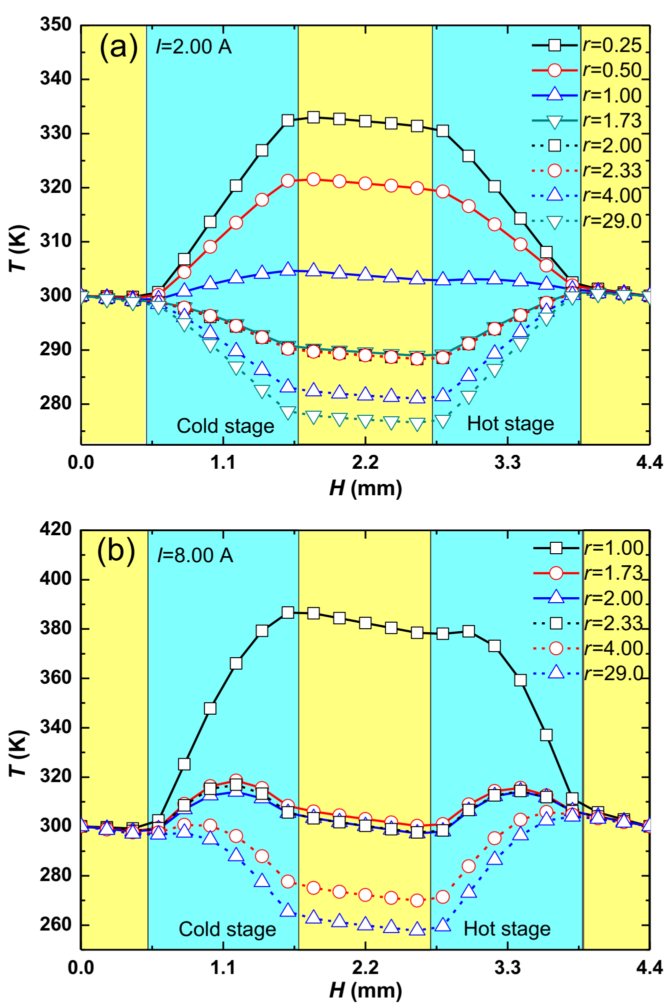

Fig. 7. Temperature distributions along height direction of the two-stage TEC for various r: (a) $I = 2.00$ A; (b) $I = 8.00$ A.

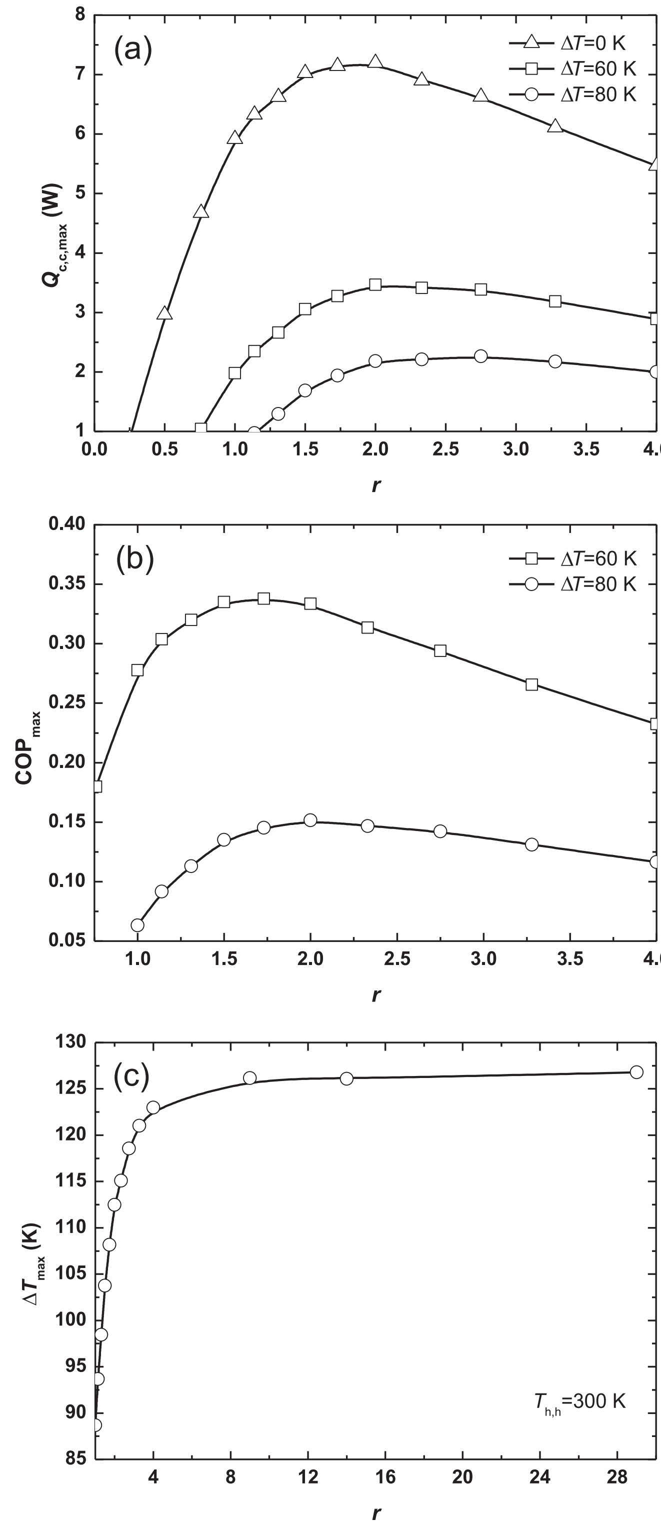

Fig. 8. The maximum cooling capacity, COP, and temperature difference of the two-stage TEC with various r: (a) cooling capacity; (b) COP; (c) maximum temperature difference.

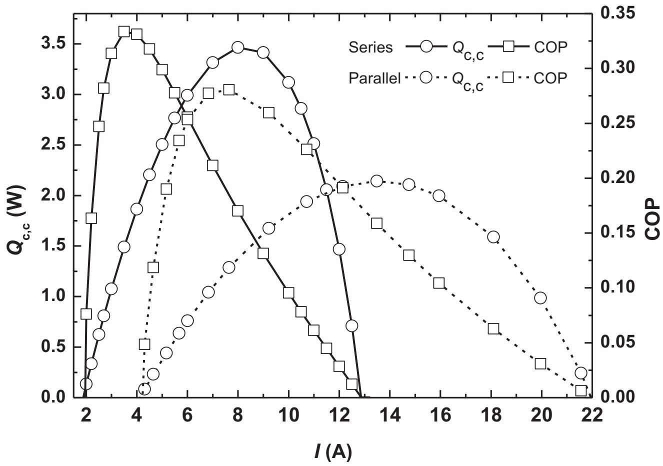

Fig. 9. Performance comparison of series and parallel configurations.

($\Delta T_c = T_{h,c} - T_{c,c}$) decreases, while the temperature difference of the hot stage ($\Delta T_h = T_{h,h} - T_{c,h}$) increases. Especially, when $r \geq 1.73$, $T_{c,c}$ becomes larger than $T_{h,c}$, and $T_{c,h}$ becomes smaller than $T_{h,h}$, because the thermoelectric element number of the hot stage exceeds that of the cold stage.

For the two-stage TEC with series configuration, the applied currents are the same for all the thermoelectric elements. The thermoelectric elements of the cold stage operate at higher temperature difference for small r, therefore, its effective working range of the applied current becomes smaller as compared with large r. Fig. 7 shows that the two-stage TEC with $r = 1.00$ has the smallest temperature differences for both the cold stage and the hot stage at $I = 2.00$ A, and its cooling capacity is the largest (Fig. 6a). Similarly, the two-stage TEC with $r = 2.00$ has the smallest temperature differences at $I = 8.00$ A, and its cooling capacity is the largest (Fig. 6a). Thus, improvement of temperature uniformity within both the cold stage and the hot stage is responsible for the enhanced cooling capacity of the two-stage TEC.

The maximum cooling capacity and COP of two-stage TEC with various r at $\Delta T = 0$, 60, and 80 K is shown in Fig. 8a and b, respectively. Similar to the one-stage TEC, $Q_{c,c,\max}$ and COP$_{\max}$ of the two-stage TEC reduces significantly as increased ΔT due to the enhanced backward Fourier heat conduction. At each ΔT, an optimal r_Q corresponding to the highest $Q_{c,c,\max}$ and an optimal r_{COP} corresponding to the highest COP are observed, and the optimal r_{COP} is always smaller than the optimal r_Q. For example, $r_{Q,\mathrm{opt}} = 2.00$ at $\Delta T = 0$ and 60 K, and $r_{Q,\mathrm{opt}} = 2.33$ at $\Delta T = 80$ K, however, $r_{\mathrm{COP,opt}} = 1.73$ at $\Delta T = 60$ K, and $r_{\mathrm{COP,opt}} = 2.00$ at $\Delta T = 80$ K. The maximum temperature difference, $\Delta T_{\max}$, produced by the two-stage TEC with different r is shown in Fig. 8c. $\Delta T_{\max}$ for each r is obtained as follows: ΔT of the two-stage TEC with the fixed r is increased until the $Q_{c,c}$–I curve reduces to a point with $Q_{c,c} = 0$. Fig. 8c shows that $\Delta T_{\max}$ monotonously increases with increased r and the increase rate becomes smaller at higher r, which agrees with the previous report predicted by the thermal resistance model [13]. For the present two-stage TEC with series configuration and $N = 30$, $\Delta T_{\max}$ is 126.80 K for $r = 29.00$, which is far higher than 70 K provided by the one-stage TEC.

4.4. Performance comparison between series and parallel configurations

The performances of the two-stage TECs with series and parallel configurations are compared under the same conditions with $N = 30$, $r = 2.00$, $T_{h,h} = 300$ K, and $\Delta T = 60$ K. Fig. 9 shows that the parallel configuration can provide a 2.14 W maximum cooling capacity and a 0.28 maximum COP, which are lower than 3.14 W and 0.33 of the series configuration. Our calculation also indicates that the parallel configuration with $r = 2.00$ can provide a 90.10 K maximum temperature, however, the series configuration can provide a 112.47 K maximum temperature.

The poor performance for the parallel configuration was also reported by Xuan et al. [11,13]. The reason can be explained as follows. For the parallel configuration, the electric potential differences of the cold and hot stages are the same, however, the applied currents are different, and $I_c > I_h$ for $r = 2$ because the thermoelectric element number of the cold stage is smaller than that of the hot stage. For the series and parallel configurations, the maximum cooling capacities occur at $I_{\mathrm{opt}} = 8.00$ and 13.49 A, respectively, and $I_{c,\mathrm{opt}} = 8.50$ A and $I_{h,\mathrm{opt}} = 4.99$ A for the parallel configuration. Fig. 10 compares the temperature distributions for the two configurations at $I = I_{\mathrm{opt}}$ ($Q_{c,c} = Q_{c,c,\max}$). For the parallel configuration, the applied current of the hot stage is only 4.99 A. In order to elevate the cooling capacity, the hot stage must increases its cold end temperature, $T_{c,h}$, to alleviate the negative effect of the backward Fourier heat conduction. Fig. 10 shows that $T_{c,h}$ for the parallel configuration ranges from 297.3 to 302.5 K, which is larger than 268.8–273.2 K for the series configuration. Thus, the hot end temperature of the cold stage, $T_{h,c}$ for the parallel configuration is also inevitably larger than that for the series configuration. Higher temperature difference of the cold stage for the parallel configuration, $\Delta T_c = T_{h,c} - T_{c,c}$, reduces its cooling capacity, hence, the parallel configuration has worse performance than the series configuration.

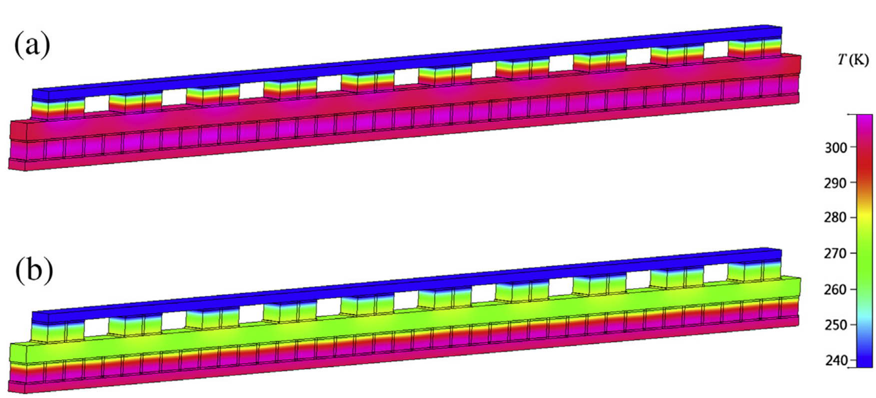

Fig. 10. Temperature distributions of the two-stage TECs with $r = 2$, $\Delta T = 60$ K, and $I = I_{\mathrm{opt}}$: (a) parallel configuration; (b) series configuration.

4.5. Optimal current ratio for separated configuration

The performance of the two-stage TEC with separated configuration is optimized for various current ratios. The total number of thermoelectric elements is $N = 30$, the number ratio is $r = 2$, the hot end temperature of the hot stage is $T_{h,h} = 300$ K, and the temperature difference is $\Delta T = 0$, 60, and 80 K. Section 4.4 has shown that for $r = 2$, when the applied current of the hot stage is smaller than that of the cold stage, the two-stage TEC has poor cooling capacity. Therefore, only $t \geq 1.0$ is considered for the separated configuration, and the separated configuration with $t = 1.0$ is equivalent to the series configuration.

The $Q_{c,c}$–COP and I_c–$Q_{c,c}$ curves of the two-stage TECs with current ratios of $t = 1.0$, 1.5, 2.0, 2.5, 3.0, and 4.0 at $\Delta T = 60$ K are shown in Fig. 11. The maximum COP monotonously decreases with the increased t. However, the maximum cooling capacity first increases and then decreases, with the optimal t ranging from 1.5 to 2.0. The effect of t on the cooling capacity can be explained by temperature distribution within the two-stage TEC. Fig. 12 shows the temperature distributions for $t = 1.0$, 1.5, and 2.0 at the same cold end current of $I_c = 8.0$ A. Increasing the hot stage current elevates the Peltier heat on its cold end, however, too high current also increases the Joule heat significantly and causes a high temperature distribution within the hot stage. Thus, increasing the hot stage current has both positive and negative effects on improving the cooling capacity. Fig. 12 indicates that $T_{h,c}$ is 267.4 K for $t = 1.5$, which is lower than 276.0 K for $t = 1.0$ and 282.5 K for $t = 2.0$. As a result, $t = 1.5$ has the lowest temperature difference, $\Delta T_c = 267.4 - 240 = 27.4$ K, for the cold stage at $I_c = 8.0$ A, and hence can provide the largest cooling capacity, and then followed by $t = 1.0$ and 2.0, as shown in Fig. 11b.

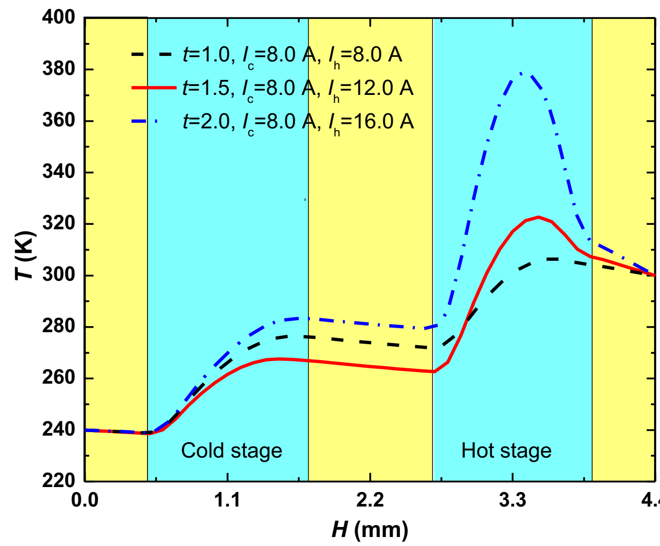

Fig. 12. Temperature distributions of separated configuration with $t = 1.0$, 1.5 and 2.0 at $I_c = 8.0$ A.

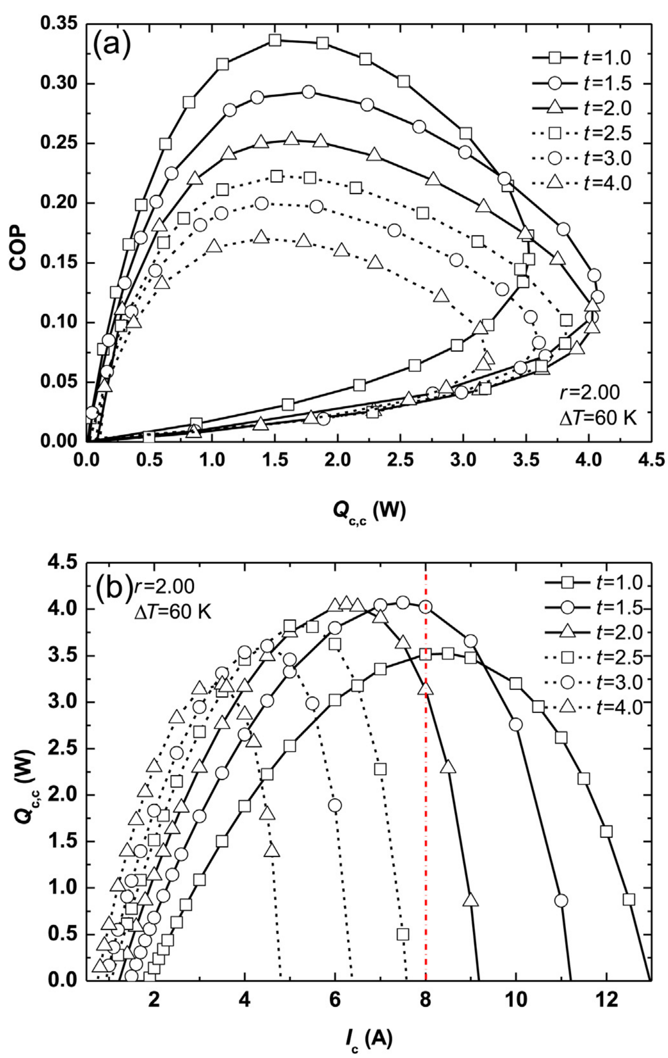

Fig. 11. Performance comparison of the two-stage TECs with different current ratios: (a) $Q_{c,c}$–COP curve; (b) I_c–$Q_{c,c}$ curve.

Fig. 13 shows the maximum cooling capacity and COP of the separated configuration with various t. With the same t, $Q_{c,c,\max}$ is the largest at $\Delta T = 0$ K, then it gradually reduces with the increased ΔT due to the enhanced backward Fourier heat conduction. For each ΔT, there exists an optimal t, the two-stage TEC has the largest $Q_{c,c,\max}$. At $\Delta T = 0$ K, the optimal t is 1.6, the corresponding $Q_{c,c,\max}$ is 7.89 W, at $\Delta T = 60$ and 80 K, t_{opt} are all 1.7, while $Q_{c,c,\max}$ is 4.10 and 2.79 W, respectively. Different from $Q_{c,c,\max}$, the largest $\text{COP}_{\max}$ always occurs at $t = 1.0$. Fig. 14 shows the maximum temperature difference of the separated configuration with various t. The maximum temperature difference first increases and then decreases as t increases, its extreme value of 120.95 K occurs for $t = 1.8$, with a 7.54% increase compared to the series configuration

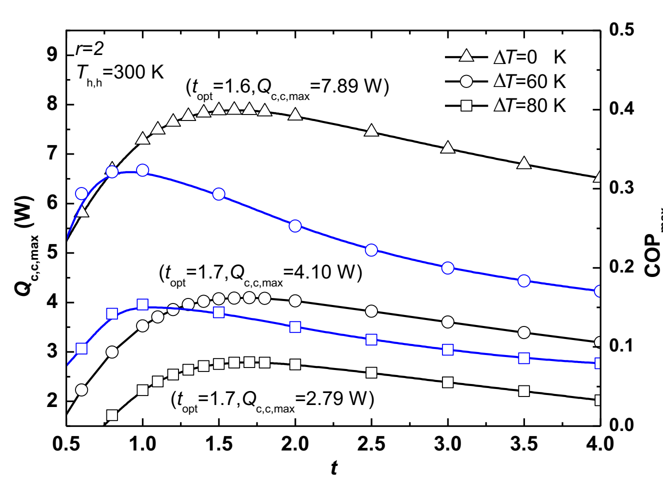

Fig. 13. Cooling capacity and COP of the separated configuration with various t.

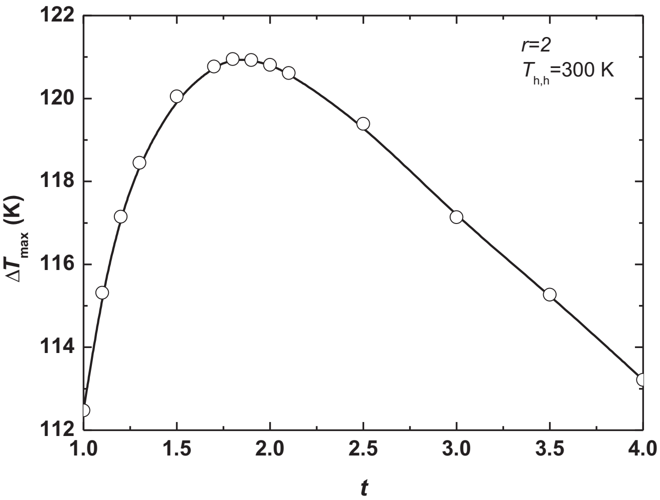

Fig. 14. Maximum temperature difference of the separated configuration with different *t*.

(t = 1.0). Thus, when the optimal number ratio, *r*, for the series configuration is obtained, the cooling capacity, COP, and maximum temperature difference of the two-stage TEC can be further improved by properly selecting the current ratio.

5. Conclusions

This work adopted a general, three-dimensional, and heat-electricity coupled model, developed by our previous studies, to investigate the performance of the two-stage TEC. Three kinds of design configurations, connected electrically in series, in parallel, and separated, were optimized. The main conclusions are as follows:

1) The arrangement of thermoelectric elements on the hot and cold stages can produce a marked three-dimensional temperature profile, when the number ratio of thermoelectric elements is not 1.00. Thus, a three-dimensional modeling is extremely necessary for accurately predicting the performance of the two-stage TEC.
2) The present results show that the constant-properties model underestimates or overestimates the cooling capacity significantly depending on the value of the current supplied to the TEC, due to inaccurate predictions of the Joule heat, Thomson heat, and Fourier heat conduction.
3) With the same number ratio, the series configuration provides a superior performance over the parallel configuration. The poor performance can be attributed to the fact that the currents supplied to the hot and cold stages for the parallel configuration are inverse proportional to their thermoelectric element numbers. However, this current distribution generally causes a very high temperature difference on the stage with small thermoelectric element number, so that the cooling capacity of this stage reduces significantly. For the series configuration, the optimal performance requires more thermoelectric elements distributed on the hot stages, because the cooling capacity of the hot stage is always larger than that of the cold stage.
4) The cooling capacity, COP, and maximum temperature difference of the two-stage TEC can be further improved by changing the series configuration to the separated configuration. However, this approach requires a higher current supplied to the hot stage. For the two-stage TEC with 30 thermoelectric elements, when the number ratio is 2.00 and the current ratio is 1.8, the maximum temperature difference is found to be 120.95 K, far higher than 70 K provided by one-stage TEC.

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